28 KSME Journal, Vol. 5, No. 1, pp. 28~35, 1991

DROPLET INTERACTION OF ADIABATIC DENSE SPRAYS

Suk Ho Lee* and Hho-Jung Kim*

(Received November 6, 1990)

The effect of the droplet interactions which arise in the interior of dense sprays is examined by considering the vaporization of a single droplet in a confined region or "bubble". In the present study the temporal variation of vaporization was determined at two levels of approximation, such as "film analysis" and "quasi-steady analysis." Thermodynamic analysis was used to determine the final equilibrium conditions which must be satisfied by the unsteady solutions. The results of these two approximate approaches to the unsteady problem were found to be in good agreement providing support for the use of the relatively simple film theory and clearly indicated that the d^2-vaporization law cannot be applied in dense sprays. It was shown that the transition between complete and incomplete evaporation is very sensitive to the initial air temperature and to the ratio of the mass of the liquid to the air. In order to gain a preliminary indication of the effect of spray size distribution on the vaporization process the film analysis was extended to sprays with a bimodal droplet size distribution, and it was found that then the kinetics of vaporization also depends on the ratio of the mass of the smaller to that of the larger droplets.

Key Words : Droplet, Interaction, Spray, Combustion, Evaporation or Vaporization, Size Distribution, Bimodal, Film, Quasi-Steadiness

NOMENCLATURE

b : Radius of the medium or bubble
c_1, c_2, c_3 : Correlation constants
c_a : Specific heat of the air
c_l : Specific heat of the liquid
c_p : Specific of the mixtures
d : Droplet diameter
D : Mass diffusion coefficient
h : Convective heat transfer coefficient
H : Effective latent of the vaporization
k_g : Mass transfer coefficient
L : Latent heat of the vaporization
m : Droplet mass
$\dot{m}_l$: The rate of change of the droplet mass
m_{la} : Ratio of the initial mass of the liquid to the air, m_{li}/m_{ai}
m_{12} : Ratio of the initial mass of the smaller to the larger droplets, m_{1i}/m_{2i}
n : Number density of the liquid roplets
P_f : Average partial pressure of non-diffusing gas in film surrounding droplet, $(P_v+P_s)/2$
P_s : Droplet vapor pressure at the temperature of the liquid surface
P_v : Partial pressure of the droplet vapor in the surrounding air
r : Radial coordinate
r_s : Droplet raius
R : Gas constant
t : Time
T : Temperature
u : Internal energy
V : Volume
W : Molecular weight
Y_f : Fuel vapor mass fraction
Le : Lewis number, $Le=\alpha/D$
Nu : Nusselt number, $Nu=\mathrm{h}\ 2r_s/\lambda$
Sh : Sherwood number, $Sh=k_m 2r_s/D$
α : Termal diffusivity of the gas, $\alpha=\lambda/\rho_g c_p$
ρ : Density
λ : Thermal conductivity of the gas-film around the droplet

Subscripts

a : Air property
av : Averaged condition
b : Condition at the bubble surface
eq : Equilibrium condition
f : Final state or liquid fuel property
g : Mixture gas
i : Initial state
l : Liquid property
r : Reference condition
s : Droplet surface
v : Vapor property
1 : Smaller droplet
2 : Larger droplet

1. INTRODUCTION

Extensive research has been conducted on the evaporation and combustion of single droplets as reviewed by Williams (1973), Faeth (1977) and Law (1982). Typically, the droplets are considered to be in an infinite medium, or in a spray which is so dilute that vaporization will be complete. Recent-

---

*Korea Institute of Nuclear Saffty P.O. Box 16, Daeduk-Danji, Taejion 305-606, Korea

ly, both experiment and theory have shown that in many cases sufficiently dense fuel sprays many evaporate or burn as a group rather than as individual droplets. Then droplet interactions may have profound effects on the efficiency, stability, and pollution characteristic of such sprays (McCreath and Chigier, 1973). The interior of dense sprays may be sufficiently saturated that vaporization can only occur near the edge of the cloud where the spray comes into contact with the surrounding medium, and it is unlikely that the single droplet results will be applicable in this case. It is also of interest to determine the time required for a spray injected into a given medium to reach saturation in relation to other characteristic times associated with spray combustion. The effects of the droplet interactions which arise in these situations can be studied by considering the vaporization of a single droplet in a confined region or "bubble", the size of which can be related to the spray density, and this is the subject of the present study.

Various aspects of dense spray droplet interactions have been studied by a number of investigators. A simple mathematical model based on the cellular method was proposed by Zung (1967). This method consists of dividing the spray into a number of identical cells, each occupied by one droplet, reducing the problem to the consideration of a single droplet. By assuming that the vapor concentration at the edge of each cell is approximately equal to the average concentration and that there is no mass transfer between cells, an expression for the cloud lifetime for both the saturated and unsaturated cases was derived in a closed form. Law (1977) studied the unsteady droplet combustion caused by droplet heating by assuming quasi-stedy gas-phase processes and a uniform droplet temperature, and also investigated adiabatic spray vaporization by analyzing the coupled problem of the vaporization of an ensemble of droplets in a gaseous medium whose properties are continuously being modified by vaporization. A set of criteria establishing the mixture properties required for achieving complete vaporization was established.

The evaporation of a droplet inside a "bubble" was considered in a recent study by Tishkoff(1979). The gas-phase quasi-steady equations were used with approximate bubble surface boundary conditions. It was found that te initial vaporization rate of a confined drop exceeded that of the corresponding single droplet, but the subsequently vaporization was retarded by cooling and by the increased vapor concentration in the surrounding bubble volume. The bubble size critically affected evaporation since as it decreased from large values where the droplet evaporated completely to lower values, saturation occurred before completee vaporization, and the droplet radius at saturation was shown to be a decreasing function of the bubble diameter. A theory of nondilute spray evaporation taking into account the separation distance between droplets has also been developed by Bellan and Cuffel(1983). This theory was based upon the global conservation equations for the two-phase mixture and the conservation equations for a droplet evaporating in finite surroundings. In this model each drop was also isolated in a sphere of influence ; however, the model differs from both the Tishkoff's bubble and Zung's cell models because it did not require zero mass transfer between cells and because the conditions at the edge of the sphere of influence were not imposed, but computed as a consequence of the droplet interatctions. The Bellan-Cuffel model was also used to develop a criterion for complete evaporation before saturation for nondilute sprays.

The present study considers the vaporization of a single droplet in a confined, insulated medium. A number of approximations can be introduced to simplify the problem, and three levels of approximation have been investigated here. At the simplest level a purely thermodynamic analysis is adopted to determine the final equilibrium state and the conditions for complete vaporization. However, this analysis doed not provide information about the vaporization rate or the approach to the final state, but indicated only the conditions which must be satisfied by the unsteady vaporization solutions at infinite time. Film theory, which assumeds that the droplet and gas temperatures are uniform and the vaporization and heat transfere occur accross a thin film at the droplet surface, has been used to compute transient vaporization at the next level of approximation. Finally, a more complex, quasi-steady analysis has been used in which the variation of gas phase composition and temperature have been considered in determining the mass and energy transfer to the droplet surface has been at the next level of approximation. The results of these two approaches were found to be almost identical thus jusifying the use of the relatively simple film theory in more complex calculations. For a preliminary assessment of the influence of particle size distribution on vaporization, the film theory also has been used to study the vaporization of two droplets of different sizes in a bubble, representative of a spray with a bimodal size distribution. The present anaysis is similar to Zung's cellular method in the the condition of zero heat and mass transfer was imposed at the bubble surface, however, is different from his model in that the convective terms in the conservation equations are considered. The internal energy of the vapor, which was not considered in Tishkoff's model, is taken into account in the present analysis.

The results of these droplet in bubble studies are then used to evaluate the vaporization characteristics of dense sprays. The results clearly indicate that the d^2-vaporization law cannot be applied in dense sprays. It is shown that the transition between complete and incomplete evaporation is very sensitive to the initial air temperature and to the mass ratio of the liquid to the air. For bimodal sprays the evaporation characteristics also depend on the mass ratio of the smaller to the larger droplets.

2. EVAPORATION OF SINGLE DROPLET

Since the evaporation of liquied droplets in a hot gas medium is associated with a change in the heat content of the two phases, the rate of this process will depend upon the interphase rates of heat and mass transfer. As a first step in investigating transient evaporation in dense sprays the vaporization of a single droplet of intial radius r_{si} in an isulated, rigid bubble is investigated for various bubble radii b, and initial gas temperatures T_{ai}. It is assumed that at time $t=0$, the droplet at temperature T_{si} is inserted into the bubble filled with dry gas to approximate the initial conditions when a spray is first injected into a dry gaseous medium.

2.1 Thermodynamic Analysis (Approach Ⅰ)

Since the droplet under consideration is in a confined medium or bubble, the temperature of the air will decrease and that of the liquid will increase as the droplet evaporates. Thus, heat transfer rates from the air to the liquid will decrease and in the case of incomplete vaporization a final equilibrim state is reached in which the liquid and gas temperatures are equal.

For given initial values of droplet radius r_{si}, droplet temperature T_{si}, air temperature T_{ai}, and bubble radius b, the final droplet radius r_f and the final equilibrium temperature T_f were determined using the mass and energy conservation equations, and the equation of state of the liquid vapor. Conservation of mass requires that:

$$m_{ai}+m_{li}=m_{af}+m_{lf}+m_{vf} \tag{1}$$

Since the mass of the air inside the medium does not change during evaporation $m_{ai}=m_{af}=m_a$; however, some of the liquid will vaporize, and it then follows from Eq.(1) that:

$$m_{vf}=\frac{4}{3}\pi\rho_l(r_{si}^3-r_f^3) \tag{2}$$

Since the bubble is insulated it follows from energy conservation that:

$$m_a c_a T_{ai}+m_{li}c_l T_{li}=m_a\,c_a\,T_f+m_{lf}c_l T_f+m_{vf}(c_v T_f+L) \tag{3}$$

Assuming the air and liquid vapor are perfect gases, the equation of state for the liquid vapor becomes:

$$P_v V_v=m_v R_v T_a \tag{4}$$

where V_v is the volume of the liquid vapor,

$$V_v=\frac{4}{3}\pi(r_{si}^3-r^3).$$

The vapor pressure P_{vf} as a function of the final temperature T_f can be obtained from the Cox-Antoine vapor pressure correlation equation(Reid, 1977), and is given by:

$$l_n\ P_v=c_1+c_2/(T_a+c_3) \tag{5}$$

where c_1, c_2, and c_3 are correlation constants.

From Eqs. (2) ~ (5), the final droplet radius r_f and the final equilibrium temperature T_f can be obtained by trial and error.

A simple, rather approximate relation for the demarcation between complete and incomplete evaporation can be obtained for a given bubble radius and initial air temperature by assuming that the change in the energy of the air is used to vaporize the droplet completely. Then the energy and mass balance euqations become for $b^3 \gg r_{si}^3$:

$$\frac{4}{3}\pi b^3\rho_a c_a(T_{ai}-T_f)\cong\frac{4}{3}\pi r_{si}^3\rho_l L \tag{6}$$

and

$$\frac{4}{3}\pi r_{si}^3\rho_l\cong\frac{4}{3}\pi b^3\frac{P_v}{R_v T_f} \tag{7}$$

Just at the point of complete vaporization the gas vapor mixture will be completely saturated, and the vapor and air temperatures will be equal to T_f. From Eqs.(6), (7), and the vapor pressure relation (5) it follows that:

$$T_{ai}=\left(\frac{\rho_l}{\rho_a}\right)\left(\frac{r_{si}^3}{b^3}\right)\left(\frac{L}{c_a}\right)+\frac{c_2}{l_n(\rho_l R_v T_f)-l_n\left(\frac{r_{si}^3}{b^3}\right)-c_1}-c_3 \tag{8}$$

where the term $\ln(\rho_l R_v T_f)$ in Eq.(8) is nearly constant especially for the range of typical final temperatures $300 < T_f < 400$°K and is smaller than the other terms, thus is may be assumed that $\ln(\rho_l R_v T_f)$ as a constant.

Equation(8) provides a simple, approximate relation between the initial air tepmerature and the bubble radius at which evaporation will just be complete. It is interesting to note that these relations are independent of the initial droplet temperature T_{si}, and this is based on the fact that Eq.(6) is based on the assumption that the internal energy of the droplet is small compared to the heat of vaporization. It also follows that vaporization will be complete if, for a given bubble radius, the initial air temperature T_{ai} is higher than that given by Eq.(8).

The above thermodynamic analysis was used to compute the final dimensionless droplet radius $\bar{r}_f(=r_f/r_{si})$ as a function of dimensionless bubble radius $\bar{b}(=b/r_{si})$ for 100μm water droplets at an inital temperature $T_{si}=300$°K, and for $T_{ai}=600$°K, 800°K, and 100°K in Fig. 1. Similar calculations also were made for n-heptane in order to compare the results of the rpesent analysis with those of Tishkoff(1979). It can be seen that saturation occurs before complete evaporation for small $\bar{b}$ but vaporization becomes complete as $\bar{b}$ increases beyond values on the order of 20 for the parameters consid-

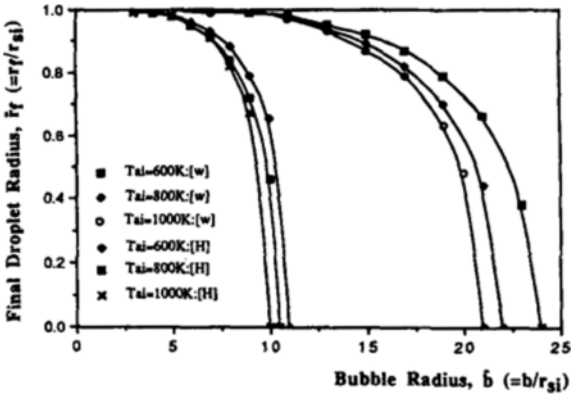

Fig. 1 Final droplet radii for water and n-heptane

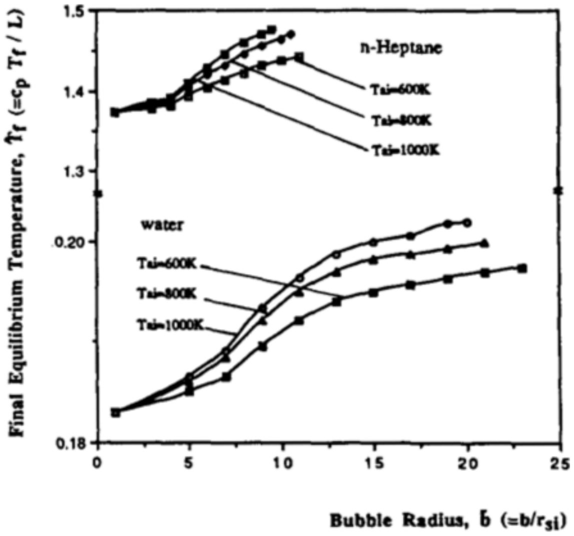

Fig. 2 Final equilibrium temperatures for water and n-heptane

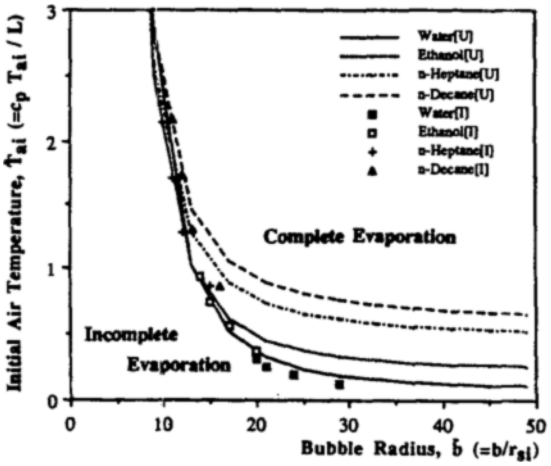

Fig. 3 Variation of $\bar{b}$ with $\bar{T}_{ai}$ at the point of complete vaporization([U] : Eq.(8), [I] : Approach I)

ered here. It is readily shown from Eqs.(3) and (4) that the variation of $\bar{r}_f$ with $\bar{b}$ is independent of the individual droplet or bubble radius but depends only on the fluid properties. The variation of the final equilibrium temperature with bubble radius and initial air temperature is shown in Fig. 2, and, as is to be expected, the final temperature approaches the initial air temperature as the bubble radius increases. The variation of dimensionless bubble radius $\bar{b}$ with the dimensionless temperature $\bar{T}_{ai}$ at the point of complete vaporization is shown in Fig. 3. The upper right side of the curves is the complete evaporation region, and the lower left side of the cruves is the incomplete evaporation region. The boundaries between complete and incomplete vaporization were computed for water, ethanol, n-heptane, and n-decane using both the exact thermodynamic theory and the approximate relation given by Eq.(8). The exact and approximate results are in excellent agreement at high values of $\bar{T}_{ai}$ where the effect of intial droplet temperature, which is neglected in the approximate theory, should be small. The exact and approximate results diverge significantly at lower values of $\bar{T}_{ai}$ where the approximate theory overestimates the bubble radius required for complete vaporization. It is interesting that the exact results for the liquids considered here all seem to fall on the same currve.

2.2 Transient Vaporization Analysis

(1) Evaporation Based on the Film Theory (Approach II)

Since the temperature and concentration changes are generally confined to a narrow boundary layer region between the liquid and gas phase, the film theory as described by Marshall (1955) can be applied here. This method, called Approach II here, is perhaps the simplest which can be sued to deal with the unsteady vaporization problem. This may be used to compute transient vaporization which was not provided in the thermodynamic analysis.

Assumptions used in the thermodynamic analysis are retained in this analysis and both the droplet and air temperatures are taken to be spatially uniform but temporally varying. Then, the rate of mass transfer from the saturated liquid surface is:

$$\dot{m}_l = -4\pi r_s^2 \rho_l \frac{dr_s}{dt} = k_g 4\pi r_s^2 (P_s - P_v) \tag{9}$$

where k_g is a mass transfer coefficent.

Energy conservation requires that the rate of heat transfer conducted across the droplet surface must supply the latent heat of vaporization and the heat required to change the droplet temperature so that:

$$\dot{m}_l L + \frac{4}{3}\pi r_s^3 \rho_l c_l \frac{dT_s}{dt} = h 4\pi r_s^3 (T_a - T_s) \tag{10}$$

Experimental studies have established empirical correlations for the heat and mass transfer coefficients, h and k_g. For the present stagnant case the Nusselt number, Nu, and the Sherwood number, Sh, were taken as 2.0 so that:

$$h = \frac{\lambda}{r_s}, \quad k_g = \frac{D\rho_a}{r_s P_f} \tag{11}$$

Since the total energy of the liquid and gas phase in the insulated bubble must remain constant, it follows that:

$$m_a c_a \frac{dT_a}{d_t} - \dot{m}_l c_l T_s + m_l c_l \frac{dT_s}{d_t} - \dot{m}_v c_v T_a + m_v c_v \frac{dT_a}{d_t} - \dot{m}_l L = 0 \tag{12}$$

Applying the Cox-Antoine vapor pressure relation (5) and the equation of state for the liquid vapor (4), three ordinary differential Eqs.(9),(10) and (12) were solved numerically with initial conditions $T_a(0) = T_{ai}$, $T_s(0) = T_{si}$, $r_s(0) = r_{si}$ for different values of the initial aire temperature T_{ai} and bubble radius b.

(2) The Quasi-Steady Analysis of Gas Phase Mass and Heat Trnasfer (Approach III)

Actually the temperature and vapor mass fraction in the atmosphere surrounding the droplet will not be uniform, and the heat and mass transfer coefficients used in the film theory provide a means of representing the effects of temperature and concentration variations in the gas which determine the heat and mass transfer to the droplet. Approach III was undertaken to investigate the importance of spatial variations in gas phase composition and temperature. Overall, as already indicated in the introduction, the bubble problem is unsteady; however, Crespo and Linan (1975) have shown that the use of the quasi-steady approximation leads to negligible error in determining the vaporization or combustion of unconfined droplets. On this basis the quasi-steady analysis was applied to the confined droplet vaporization problem under consideration here although, as will be seen below, this does lead to some difficulties in satisfying the boundary conditions at the bubble surface. Assumptions used in the thermodynamic analysis are are still assumed to be valid, and the assumption of uniform droplet temperature has been retained. Also, it is assumed that Lewis Numeber, Le, is equal to one, and that the gas phase pressure is spatially invariant.

Following Law (1980) the quasi-steady solutions for $r_s < r < b$ can be expressed in the following form:

$$Y_f = 1 - (1 - Y_{fb}) \operatorname{Exp}\left\{ -\frac{\dot{m}_l}{4\pi\rho D r_s}\left(\frac{1}{r} - \frac{1}{b}\right)\right\} \tag{13}$$

and

$$T = T_s - \frac{H}{c_p}\left\{1 - \operatorname{Exp}\left\{ -\frac{\dot{m}_l}{4\pi\rho D r_s}\left(\frac{r_s}{r} - 1\right)\right\}\right\} \tag{14}$$

where

$$H = \frac{c_p (T_b - T_s)}{\dfrac{(1 - Y_{fb})}{(1 - Y_{fs})} - 1} \tag{15}$$

$$\dot{m}_l=4\pi\rho D r_s\frac{\ln(1+B)}{1-\frac{r_s}{b}} \quad (16)$$

$$B=\frac{c_p(T_b-T_s)}{H} \quad (17)$$

H is an effective latent heat of vaporization which includes the actual latent heat of vaporization and the energy used to heat the droplet interior per unit mass of fuel vaporized, and is denfined by the equation below:

$$4\pi r_s^2\lambda\frac{dT}{dr}\Big|_{r=rs}=\dot{m}_l H=\dot{m}lL+\frac{4}{3}\pi r_s^3\rho_l c_l\frac{dT_s}{dt} \quad (18)$$

It is assumed that the vapor and the liquid are in equilibrium with each other.

Hence, the fuel vapor mass fraction at the droplet surface is calculated from

$$\frac{P_v}{P}=\frac{Y_{fs}/W}{Y_{fs}/W+(1-Y_{fs})/W_a} \quad (19)$$

Note that while H depends on the derivative dT_s/dt, the quasi-steady solution above treats T_s and T_b as constants. The variation of T_s and T_b with time can be determined only by considering overall energy conservation as described below. Another difficulty with the quasi-steady solution is that the temperature gradient at the bubble surface remains non-zero after all permissible boundary conditions have been applied, and this is inconsistent with the adiabatic boundary condition there.

In order to deal with these difficulties it is necessary to satisfy overall energy conservation at each time step of the quasi-steady calcualtion. The quasi-steady solution shows that there is a very steep temperature and mass fraction gradient at the droplet surface as long as r_s/b remains small. Hence it is reasonable to assume that the gas temperature outside the droplet is constant and equal to $T_b(t)$, the bubble surface temperature, and to equate $T_b(t)$ to an average gas temperature $T_{av}(t)$ determined from the relation:

$$T_{av}=\frac{\int_{rs}^{b}4\pi r^2\rho c_p T(r,t)\,dr}{\frac{4}{3}\pi\rho c_p(b^3-r_s^3)}\cong T_b(t) \quad (20)$$

A similar argument applies to the vapor mass fraction so that:

$$Y_f(b,t)=Y_b(t)\cong Y_{av}(t) \quad (21)$$

It is now possible to compensate for the unsteady effects at each time step by computing the temporal variation of the liquid and gas temperatures $T_s(t)$and $T_b(t)$ and the vapor mass fraction $Y_b(t)$ from the equations for the conservation of the total energy and mass. The conservation of the total energy at any same as in Eq.(12). Similarly, it follows from the conservation of the total mass of liquid plus vapor that:

$$Y_{fb}=\frac{m_v}{m_a+a_v}=\frac{r_{si}^3-r_s^3}{\frac{\rho_a}{\rho_l}(b^3-r_{si}^3)+(r_{si}^3-r_s^3)} \quad (22)$$

In his study of the droplet in a bubble Tishkoff(1979) used the following approximate energy conservation equation to determine the variation of the bubble surface temperature $T_b(t)$:

$$m_a c_p\frac{dT_b}{dt}=-m_l L-m_l c_l\frac{dT_s}{dt}$$

This relation neglects the energy of the vapor which, however is included in Eq. (12) used in the present calculations. As shown below Ticknoff's approximation has a significant effect of the results.

From the initial conditions $T_a(r,0)=T_{ai}$, $T_s(0)=T_{si}$, $r_s(0)=r_{si}$, the mass fractions Y_{tb} and Y_{fs} can be evaluted using the 4th order Runge-Kutta Method.

Figs. 4 and 5 show the effects of the bubble radius and the initial air temperature on transient evaporation. It can be seen that the droplet radius and the temperatures of the air and liquid at first change rapidly but then approach the equilibrium state quite slowly. Calculations were made using both approaches Ⅱ and Ⅲ and the results were found to be in excellent agreement providing justification for the use of the simpler flim theory, at least for the range of parameters considered here. It was also found that the final droplet

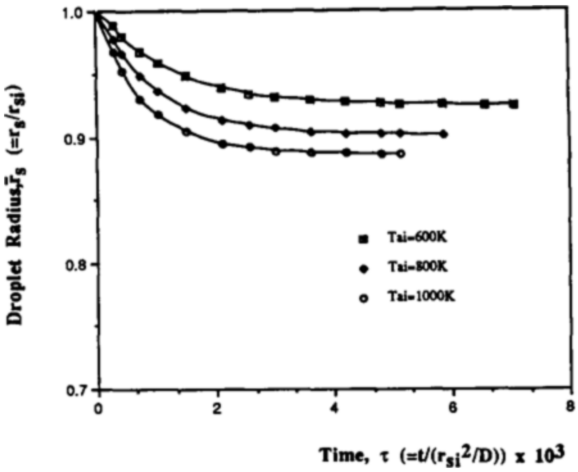

Fig. 4 Effect of Initial air temperature for water, $\bar{b}=15$

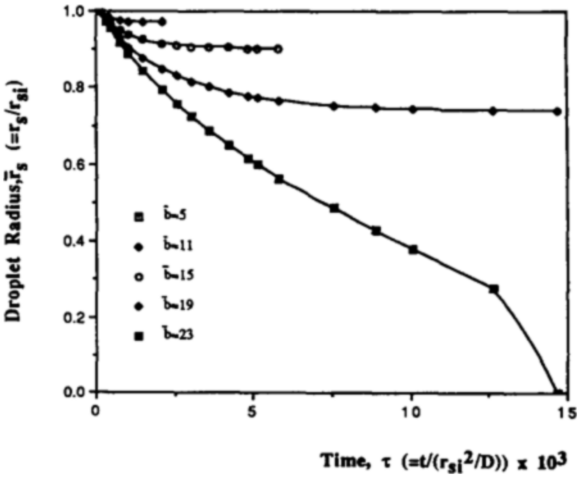

Fig. 5 Effect of bubble radius for water, $T_{ai}=800$°K

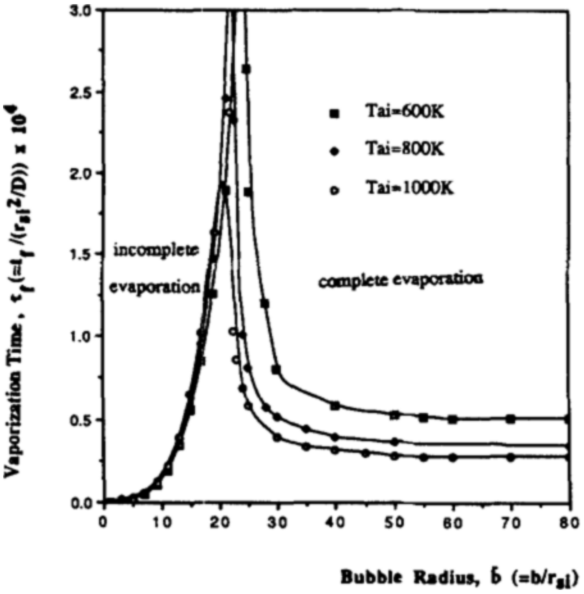

Fig. 6 Effect of bubble radius on droplet vaporization for water

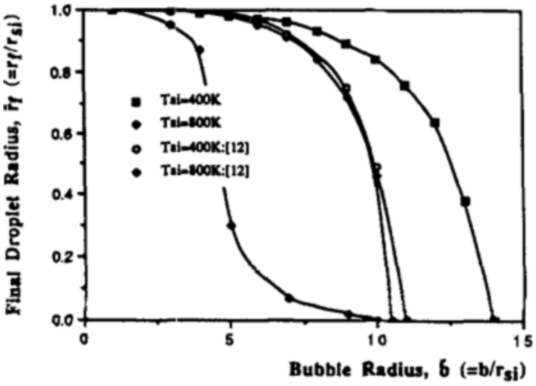

Fig. 7 Comparison with tishkoff's results(1979) for n-heptane

radius and temperature computed using the transient analysis were in excellent agreement with the values computed using the thermodynamic analysis (Approach I) which were shown in Fig. 1 and Fig. 2. Of course, to be consistent, the transient results must approach the final equilibrium values computed from steady state thermodynamic considerations. It is interesting to note from Fig. 5 that the variation of droplet radius with time becomes very sensitive to the dimensionless bubble radius $\bar{b}$ near the point of complete evaporation.

The dimensionless time for both incomplete and complete evaporation is shown in Fig. 6 for water droplets with $T_{ai}=800$°K, and $T_{si}=300$°K. Here the time has been made dimensionless using the characteristic diffusion time as the reference. The time τ_f to reach equilibrium increases with increasing $\bar{b}$ until $\bar{b}$ reaches the value for complete evaporation. After that the τ_f decreases to the vaporization time of a droplet in an infinite medium. This behavior reflects the fact that for $\bar{b}$ near unity very little vaporization occurs; however, near complete vaporization the rate is reduced by the relatively high vapor concentration within the bubble. The variation of final droplet radius with bubble radius as computed in the present analysis and by Tishkoff(1979) is shown in Fig. 7, and it can be seen that the results of the present study differ from those of Tishkoff because of the differences in the approximate bubble surface boundary conditons as already mentioned above.

3. EVAPORATION OF BIMODAL SPRAYS

A knowledge of the interactive effect of sprays with generalized droplet-size distribution is important in determining the spray vaporization time and other evaporation characteristics. In order to provide a prelimiary indication of how polydisperse sprays evaporate without the complexity involved in dealing with the continuous distributions the results of the droplet in bubble analysis have been applied to the case of the vaporization of a bimodal spray.

A key result of the analysis of single droplet case is that the film theory(Approach II) can be used to determine droplet vaporization provided that the diemensionless bubble radius $\bar{b}$ is not too close to 1, or correspondingly that the droplet numnber density is not excessive. Assuming this to be the case, the bimodal spray is treated as an assemblage of small droplets of radius r_{sl} with number density n_l and correspondingly large droplets of radius r_{s2} with number density n_2. The gas vapor mixture between the droplets is taken to be uniform and heat and mass transfer occurs across a very thin film at the droplet surface.

Using the film theory the rate of mass transfer from the saturated liquid surface for the smaller droplets and the larger droplets, respectively, now becomes

$$\dot{m}_{11}=-4\pi n_1 r_{s1}{}^2\rho_1\frac{dr_{s1}}{dt}=k_{g1}4\pi n_1 r_{s1}^2(P_{s1}-P_v) \tag{23}$$

$$\dot{m}_{12}=-4\pi n_2 r_{s2}{}^2\rho_1\frac{dr_{s2}}{dt}=k_{g2}4\pi n_2 r_{s2}^2(P_{s2}-P_v) \tag{24}$$

Also, energy conservation equation at the droplet surface when applied to both the smaller and larger droplets yields the relations.

$$\dot{m}_{11}L+\frac{4}{3}\pi n_1 r_{sl}{}^3\rho_1 c_1\frac{dT_{s1}}{dt}=h_1 4\pi n_1 r_{s1}{}^2(T_a-T_{s1}) \tag{25}$$

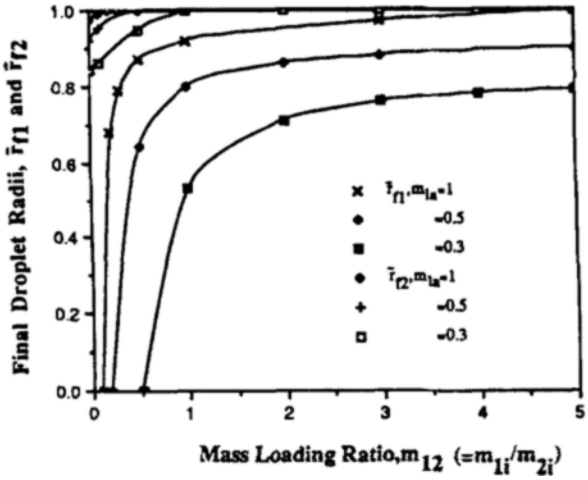

Fig. 8 Effect of m_{12} for bimodal water droplets at $T_{ai}=600$°K

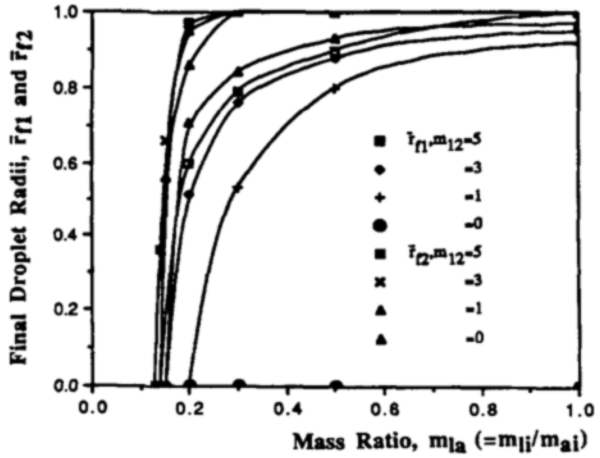

Fig. 9 Effect of m_{la} for bimodal water droplets at $T_{ai}=600°K$

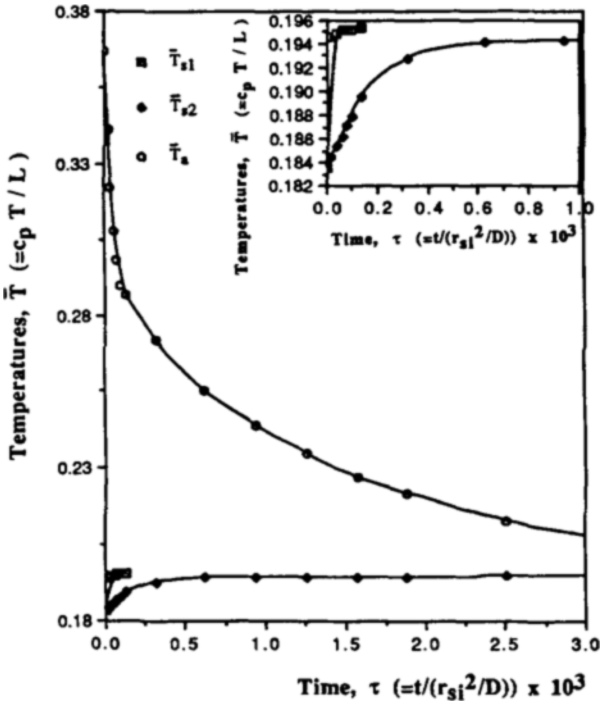

Fig. 10 Temperature variations for $m_{la}=0.5$, and $m_{12}=0.1$ at $T_{ai}=600°K$ and $T_{si}=300°K$

$$\dot{m}_{12}L+\frac{4}{3}\pi n_2 r_{s2}^2\rho_1 c_1\frac{dT_{s2}}{dt}=h_2 4\pi n_2 r_{s2}^2(T_a-T_{s2}) \qquad (26)$$

Applying the conservation of the total energy, we obtain

$$m_a c_a\frac{dT_a}{dt}-\dot{m}_{11}c_1 T_{s1}-\dot{m}_{12}c_1 T_{s2}+m_{11}c_1\frac{dT_{s1}}{dt}+m_{12}c_1\frac{dT_{s2}}{dt}+\dot{m}_v c_v T_a+m_v c_v\frac{dT_a}{dt}+(\dot{m}_{11}+\dot{m}_{12})L=0 \qquad (27)$$

With the given initial conditons n_1, n_2, T_{ai}, $T_{s1i}=T_{s2i}$, $r_{s1}=r_{s1i}$, $r_{s2}=r_{s2i}$, there are five unknows $r_{s1}(t)$, $r_{s2}(t)$, $T_a(t)$, $T_{s1}(t)$ and $T_{s2}(t)$ with five Eqs.(23, 24, 25, 26, 27). These ordinary differential equations have been solved using the 4th order Runge-Kutta Method.

The results have been evaluated in terms of the ratio of the initial mass of the liquid to the air (defined as "mass ratio"), m_{la}, or the ratio of the initial mas of the smaller to the larger droplets (defined as "mass loading ratio"), m_{12}. Calculations have been performed for a bimodal spray with 10μm and 100 μm water droplets at an initial temperature 300°K. Th effects of mass ratio and mass loading ratio are displayed in Figs. 8 and 9 which show that smaller droplets evaporate faster than larger droplets and both smaller and larger droplets evaporate completely as the mass ratio m_{la} or the mass loading ratio m_{12} decrease. A typical example of the temperature variations during transient evaporation of the bimodal spray above are shown in Fig. 10 for $m_{la}=0.5$, $m_{12}=0.1$ and $T_{ai}=600°K$. Both the liquid and air temperatures initially change rapidly and then slowly approach the equilibrium state. Particularly, due to the smaller droplet surface area, the temperature of the smaller droplets increases much faster thatn that of the lareger droplets and they evaporate faster than the larger droplets because of their larger surface to volume ratio.

4. CONCLUSIONS

A model for the evaporation of a single droplet in an adiabatic bubble has been developed. In considering transient evaporation it has been shown that the relatively simple film theory provides sufficient accuracy as long as the spray is not too dense. The results of the droplet in bubble calculations have been applied to the determination of the evaporation characteristics of monodisperse sprays. For a single droplet with a small dimensionless bubble radius $\bar{b}$, decrease in the initial air temperature $\bar{T}_{ai}$ wil cause transition from complete to incomplete evaporation and the conditions for the transition have been established. The analysis was extended to sprays with a bimodal size distribution and the ratio of the mass of the liquid to the air, m_{la}, and of the mass of the smaller to the larger droplets, m_{12}, was found to have a large effect on vaporization charactereistics of the cloud.

REFERENCES

Bellan, J. and Cuffel, R., 1983, "A Theory of Non-dilute Spray Evaporation Based Upon Multiple Drop Interactions," Combustion and Flame, Vol. 51, pp. 55~67.

Crespo, A. and Linãn, A., 1975, "Unstead Effects in Droplet Evaporation and Combustion," Combust. Sci., Tech., Vol. 11, pp. 9~18.

Faeth, G.M., 1977, "Current Status of Droplet and Liquid Combustion," Prog. Energy Combust. Sci., Vol. 3, pp .191 ~224.

Law, C.K., 1977, "Adiabatic Spray Vaporization With Droplet Temperature Transient," Combust. Sci. Tech., Vol. 15, pp. 65~74.

Law, C.K., Chung, S.H. and Srinivasan, N., 1980, "Gas-Phase Quasi-Steadiness and Fuel Vapor Accumulation Effects in Droplet Burning," Combustion and Flame, Vol. 38, pp. 173~198.

Law, C.K., 1982, "Recent Advances in Droplet Vaporization and Combustion," Prog. Energy Combust. Sci., Vol. 8, pp .171

~224.

Marshall, Jr., W.R., 1955, "Heat and Mass Transfer in Spray Drying," Trans. ASME, pp. 1377~1385.

McGreath, C.G. and Chigier, N.A., 1978, "Liquid Spray Burning in the Wake of Stabilized Disk," Fourteenth Symposium (International) on Combustion, The Combustion Institute, pp. 1355~1363.

Reid, R.C. et al., 1977, The properties of gases and liquids, 3rd ed., McGraw Hill Book Company.

Tishkoff, J.M., 1979, "A Model For the Effect of Droplet Interactions an Vaporization," International Journal of Heat and Mass Transfer, Vol. 22, pp. 1407~1415.

Williams, A., 1973, "Combustion of Droplets of Liquid Fuels: A Review," Combustion and Flame, Vol. 21, pp. 1~31.

Zung, J.T., 1967, "Evaporation Rate and Lifetimes of Clouds and Sprays in Air-The Cellular Model," J. Chem. Phys. Vol. 46, pp. 2064~2070.